A model of the prey counterattack with delay

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There has been a fair amount of previous work on stage-structured models of population growth consisting of immature and mature individuals for single species, where the stage-structure is modeled by the introduction of a constant time delay (see, for example, [1]). It is observed that for predatory plankton and fish, there exists the case that the mature fish is predatory to the plankton but the immature one is the prey of the plankton.

In the present work we consider a delay differential system of the form

$$\begin{aligned} x'(t) &= x(t)[r_1 - a_{11}x(t) + \alpha y(t) - a_{13}Y(t)] \\ y'(t) &= y(t)[-r_2 - \alpha x(t)] + a_{31}x(t)Y(t) \\ &\quad -a_{31}x(t-\tau)Y(t-\tau)\exp\left\{\int_{t-\tau}^{t}[-r_2 - \alpha x(s)ds]\right\} \\ Y'(t) &= -r_3Y^2(t) + a_{31}x(t-\tau)Y(t-\tau)\exp\left\{\int_{t-\tau}^{t}[-r_2 - \alpha x(s)ds]\right\}. \end{aligned} \tag{1}$$

Here α is a nonnegative constant and all the rest of parameters are positive. y and Y denote, respectively, the densities of immature and mature populations for single species where τ represents a constant time to maturity. x is the prey of Y but eats y, which we may call *the prey counterattack with time delay*.

We believe that this is the first time such a population model has appeared in the literature. Our purpose is to discuss qualitative properties of the model (1).

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References

[1] Aiello, W.G. & H.I. Freedman, 1990, A time-delay model of single-species growth with stage structure, *Math. Biosci*, 101, 139-153.