Boundedness and Almost Periodicity for Some State-Dependent Delay Differential Equations

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Differential delay equations or functional differential equations have been used in the modelling of scientific phenomena for many years. Often it has been assumed that the delay is either fixed constant or given as an integral which is called the distributed delay. However, in recent years, the more complicated situation in which the delay depends on the unknown functions has been proposed in models. These equations are frequently called equations with state-dependent delay. There are many works related to this topics, see the references therein.

In this work we study the existence of bounded, periodic and almost periodic solutions of the state-dependent delay differential of the following form

$$\begin{cases} \dfrac{d}{dt}x(t) = F(t, x(t), x(t-\rho(x_t))), \text{ for} t \geq 0 \\ x_0 = \varphi \in C = C([-\tau, 0]\,; \mathbb{R}^n) \end{cases} \tag{1}$$

where C is the space of all continuous functions from $[-\tau, 0]$ into R^n endowed with the uniform norm topology. For every $t \geq 0$, the history function $x_t \in C$ is defined by

$$x_t(\theta) = x(t+\theta), \text{ for } \theta \in [-\tau, 0]\,.$$

F is a continuous function from $R \times R^n \times R^n$ into R^n and ρ is a positive bounded continuous function on C, τ is the maximal delay that is defined by

$$\tau = \sup_{\varphi \in C} \rho(\varphi).$$

According to the book of Hale [?], it's well known that if F is continuous, then equation (**??**) has at least one maximal solution $x(., \varphi)$ which is defined on some interval $[0, t_\varphi)$ and if t_φ is finite then:

$$\overline{\lim_{t \to t_\varphi}} |x(t, \varphi)| = \infty.$$

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The uniqueness may not hold here, because the right hand side of (**??**) is not locally lipschitz. Even if F is lipschitzian with respect to the second or the third arguments, uniqueness may not hold:

Recently, differential equations with state-dependent delay have been the subject for several works. In [?] the author proved the existence and periodicity for some state-dependent delay differential equation. In [?] it has been proved also the existence of oscillatory and periodic solutions for some state dependent delay differential equations arising from population dynamic problems. In [?], it has been proved the stability of some state-dependent model arising form epidemic problem.

The organization of this work is as follows: in section 2 we recall some preliminaries results about ordinary differential equations that will be used in the work. In section 3, we study the problem of the existence of bounded and almost periodic solutions of equation (**??**). The remaining section is devoted to some example.

References

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