

Age-dependent structured population models and their numerical solution

Luis M. Abia¹, J.C. López-Marcos² and Oscar Angulo³.

We review some of the most significant numerical methods considered along the last two decades for the numerical integration of continuous time formulations of age structured population models, discussing their main features: what kind of problems they solve, their complexity and order of convergence, and if there is or not a convergence analysis for them. The most general model problem we consider takes the form

$$u_t(a, t) + u_a(a, t) = -\mu(a, I_\mu(t), t) u(a, t), \quad 0 < a < A, \quad t > 0, \quad (1)$$

$$u(0, t) = \int_0^A \alpha(a, I_\alpha(t), t) u(a, t) da, \quad t > 0, \quad (2)$$

$$u(a, 0) = \phi(a), \quad 0 \leq a \leq A. \quad (3)$$

where the independent variables a and t denote, respectively, age and time, and the function $u(a, t)$ is the population density function at time t with respect to age of the individuals. The dynamic of the population is determined by the mortality rate function μ and the fertility rate function α , both nonnegative functions depending on age, time, and respectively the weighted averaged quantities

$$I_\mu(t) = \int_0^A \gamma_\mu(a) u(a, t) da, \quad I_\alpha(t) = \int_0^A \gamma_\alpha(a) u(a, t) da, \quad t > 0. \quad (4)$$

¹Departamento de Matemática Aplicada y Computación, Facultad de Ciencias, Universidad de Valladolid, C/ Prado de la Magdalena s/n, 47005 Valladolid, Spain (e-mail: abia@mac.cie.uva.es).

²Departamento de Matemática Aplicada y Computación, Facultad de Ciencias, Universidad de Valladolid, Valladolid, Spain (e-mail: lopezmar@cpd.uva.es).

³Departamento de Matemática Aplicada a la Técnica, Escuela Universitaria Politécnica, Universidad de Valladolid, C/ Francisco Mendizábal 1, 47014 Valladolid, Spain (e-mail: oscar@gauss.mat.eup.uva.es).

When $\gamma_\mu(a) = \gamma_\alpha(a) = 1$, $a \in [0, A]$, we get the nonautonomous Gurtin-MacCamy model with finite age. The monographs of Iannelli [1] and Cushing [2] provide a detailed analysis of the autonomous case with the vital functions depending on an arbitrary number of functionals like (4).

References

- [1] M. Iannelli, *Mathematical Theory of Age-Structured Population Dynamics*, Applied Mathematics Monographs, 7, Consiglio Nazionale delle Ricerche, Pisa, 1995.
- [2] J. M. Cushing, *An Introduction to Structured Populations Dynamics*, CMB-NSF Regional Conference Series in Applied Mathematics, SIAM, Philadelphia, PA, 1998