The Stabilizability of a Controlled System Describing the Dynamics of a Fishery

Rachid Mchich [1] , Pierre Auger[2] and Nadia Raissi[3].

This work presents a stock-effort dynamical model, which is a set of four ordinary differential equations. It describes the evolution of two populations growing and moving between two fishing zones, on which they are harvested by two different fleets. In equations of the fishing effort variations, a control function is introduced as a rate of the revenue investment for each fleet.

The complete system reads as follows:

$$\begin{cases} \dot{x}_1(t) = R\,(kx_2 - k'x_1) + [r_1x_1(1 - \dfrac{x_1}{K_1}) - q_1E_1x_1] \\ \dot{x}_2(t) = R\,(k'x_1 - kx_2) + [r_2x_2(1 - \dfrac{x_2}{K_2}) - q_2E_2x_2] \\ \dot{E}_1(t) = R\,[mE_2 - m'E_1] + \alpha(t)E_1(p_1q_1x_1 - c_1) \\ \dot{E}_2(t) = R\,[m'E_1 - mE_2] + \alpha(t)E_2(p_2q_2x_2 - c_2) \end{cases} \tag{1}$$

The function $\alpha(t)$ is regarded as an investment rate (with respect to time) of the fishing revenue. We assume that: $-1 \preceq \alpha(t) \preceq 1$. A negative investment can be seen as a reduction of the fishing effort.

The analysis of the stabilizability for the aggregated model, in the neighborhood of the interesting equilibrium point, enables the determination of a Lyapunov function , which ensures the existence of stabilizing discontinuous feedback of the system. This enables to control the system and to lead, in an uniform way, any solution of the system towards the equilibrium point.

[1]Labo. SIANO, dept. maths & info., B.P. 133, Kénitra, Morocco (e-mail: racmchich@yahoo.com).
[2]UMR CNRS 5558 Laboratoire de Biométrie, Génétique et Biologie des Populations. Université Claude Bernard Lyon1, 43, boulevard du 11 Novembre 1918 69622 Villeurbanne cedex, FRANCE (e-mail: pauger@biomserv.univ-lyon1.fr).
[3]Labo. SIANO, dept. maths & info., B.P. 133, Kenitra, Morocco (e-mail: n.raissi@mailcity.com).

References

[1] Auger, P.M. et Poggiale, J.C., 1996. Emergence of Population Growth Models: Fast Migration and Slow Growth. Journal of theoretical Biology, 182: 99-108.

[2] Clarke, F.H., Ledyaev, YU.S., Strem, R.J. et Wolenski, P.R., 1998. Nonsmooth Analysis and Optimal Control Theory, Graduate Texts in Mathematics-Springer 178.

[3] Mchich, R., Auger, P.M. et Raïssi, N., 2000. The Dynamics of a Fish Stock Exploited Between Two Fishing Zones. Acta Biotheoretica. Vol. 48, No. 3/4, pp. 207-218.

[4] Mchich, R., Auger, P.M., de la Parra, R.B. et Raïssi, N., 2001. Dynamics of a Fishery on Two Fishing Zones with Fish Stock Dependent Migrations: Aggregation and Control. Ecological Modelling, Vol. 158, Issue 1-2, pp. 51-62.

[5] Poggiale, J.C., 1994. Applications des Variétés Invariantes à la Modélisation de l'Hétérogénéité en Dynamique des Populations. PhD thesis at Bourgogne University, Dijon.