The effect of time delay on reaction-diffusion fronts modelling biological invasions

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Models including diffusion (it is, random motion) and reaction (production of new particles) terms have been widely applied to the study of biological invasions since the pioneering work by Fisher [1]. The advantage of such kind of models is that in general they behave as propagative processes, where the speed of the travelling front derived can be taken as the population spread rate. Specifically, the FKPP-equation

$$\frac{\partial n}{\partial t} = D\frac{\partial^2 n}{\partial r^2} + F(n) \qquad (1)$$

is known to lead to fronts which travel at Fisher's speed

$$v_{Fis} = 2\sqrt{aD}. \qquad (2)$$

In equation (1) n denotes the density of particles (individuals) in position r at time t, D is the diffusion coefficent and $F(n) = an(1-n)$ is a logistic reaction term, with a the initial growth rate. Neverthless, this approach poses some theoretical problems, as it does not take into account delay effects [4]. In biological terms, they are due to the nonvanishing time interval between successive generations [2]. When these effects are considered, a time delay must be introduced in the diffusion process and Eqs. (1) and (2) turn into [2,3]

$$\frac{T}{2}\frac{\partial n}{\partial t} + \frac{\partial n}{\partial t} = D\frac{\partial^2 n}{\partial r^2} + F(n) + \frac{T}{2}\frac{\partial F(n)}{\partial t} \qquad (3)$$

$$v = \frac{2\sqrt{aD}}{1 + a\frac{T}{2}}, \qquad (4)$$

where T is the time between successive jumps (esentially, the mean generation time in biological applications). Eq. (3) is known as the hyperbolic reaction diffusion (HRD) equation. The hyperbolic front rate in expression (4) is the main result here as it generalizes Fisher's speed, which is recovered in the classical case when considering instantaneous jumps ($T = 0$). In this work we give experimental support to the hypothesis that HRD model may serve to predict expansion rates in biological systems better than classical (non-delayed) equation. Data and comparisons relative to several biological invasions (and human range expansions) are provided.

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