

The Effect Of Seasonal Host Birth Rates On Population Dynamics: A Max/Min Approach

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It is important to understand how diseases interact with their host, how they persist and, conversely, how they can be controlled. This is vital for both public health and economic reasons. Mathematical models have been used very successfully to give insights into important disease systems such as BSE and HIV/AIDS. However, many of the simple mathematical models currently in use often fail to capture important biological factors. We extend current models of host-pathogen interactions to include seasonality in the birth rate. The model considered is based on the SIR Models of Anderson and May. We are interested what happens to the dynamics of the population when the birth rate, a , is seasonal. In other words, we allow a to be a function which varies over time. Many models of childhood diseases have assumed that seasonal forcing could be modelled by a periodic function (Keeling et al), therefore, we have chosen to incorporate a sine wave into the birth rate for simplicity. The function is then bounded so we look at maximum/minimum theory which holds for some diseases, but in other diseases more complex dynamics occur. The general effects of seasonality on generic disease systems will be discussed - for example, Cowpox in Bank Voles, Fox Rabies in Europe and Rabbit Haemorrhagic Disease.

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