Spatiotemporal Propagation of FIV within Populations of Cats

Mathematical Analysis and Numerical Study of Infection Waves

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The spatiotemporal propagation of the Feline Immunodeficiency Virus (FIV) within populations of domestic cats, *Felis catus*, is analyzed using a simple continuous and deterministic SI reaction-diffusion model. As incidence terms, proportionate mixing as well as mass action are used. Demography is either of the logistic type or exhibits an Allee effect, but also constantly increasing populations as well as the situation where demography is ignored, are considered as limiting cases.

As a result of the mathematical analysis of the local dynamics, the system will finally reach an endemic stationary state, the total population will go extinct, or the eradication of disease will occur. However, periodic solutions do not exist and thus oscillations and waves do not appear.

The system behaviour depends crucially on the transmission rate of the FI Virus. By varying this parameter, the spatial spread of infection waves is numerically studied. Considering simple logistic growth, different speeds as well as contrary main directions of spatial spread can be found, depending just on both intrinsic properties and initial conditions. Moreover, boosts of infection at the head of the propagation front may divide the uniform endemic pattern temporally.

In the case of an Allee effect in population growth, further spatiotemporal patterns appear. In homogeneous environment, transient patchy infection structures appear before ultimate extinction. In heterogeneous environments, the prevalence rate of infection can be reinforced by population interaction. Furthermore, spatially critical size effects due to the bistability allow the derivation of a reduced system, in which the minimum viable spatial size of a population patch is analyzed. This may have important applications, e.g. for containment of F.I.V. on the one hand and for immunocontraception of invasive cats on the other hand.

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