Potential-growth indicator problem in matrix models of population dynamics

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Given a set of demographic parameters for a structured model population, the potential-growth indicator, R, is a calculable indicator of growth/decline/equilibrium in the population size. Calculating R enables one to see the asymptotic dynamics without finding the dominant eigenvalue, λ_1, of the projection matrix.

It is of applied value whenever available data are insufficient to calibrate the projection matrix. It is long-known that if, in the classic Leslie model, $p(\lambda)$ denotes the characteristic polynomial, then $R = 1 - p(1)$ is quite easy to calculate [1]:

$$R = b_1 + \sum_{i=2}^{n} b_i s_1 \dots s_{i-1},$$

where b_i and s_i are age-specific birth and survival rates, and $R >/</= 1$ is equivalent respectively to $\lambda_1 >/</= 1$. In the Lefkovitch model for stage-structured population, the same equivalence was proved [2] to hold for

$$R = 1 - \prod_{j=1}^{n}(1 - r_j) + \sum_{i=1}^{n+1} l_i b_i \prod_{j=i+1}^{n}(1 - r_j),$$

where formally $r_{n+1} = 0$, $l_1 = 1$, $l_i = s_1 s_2 \dots s_{i-1}$.

In Logofet's models for age-stage-structured populations, the rojection

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matrix expands the Lefkovitch one up to the following general form:

$$L = \begin{bmatrix} b_1 + r_1 & b_2 & \cdots & b_{n-1} & b_n \\ a_{21} & r_2 & & & \\ a_{31} & a_{32} & \ddots & 0 & \\ & \vdots & \ddots & & \\ a_{n-1,1} & a_{n-1,2} & & r_{n-1} & \\ a_{n,1} & a_{n,2} & \cdots & a_{n,n-1} & r_n \end{bmatrix},$$

with nonnegative low-triangular entries. Established is the following

Potential-Growth Indicator Theorem:

If L is an indecomposable Logofet matrix with nonnegative elements and a_{pq},$r_i \le 1$, then

$$R = 1 - (1 - r_1 - b_1)\prod_{h=2}^{n}(1 - r_h) + \sum_{i=0 \dots n-2; j=n-i \dots n; \sigma} b_j s_j^{n-1-i} \prod_{h \neq j, \sigma}(1 - r_h)$$

with σ meaning all possible non increasing permutation of indexes in product of a_{pq} is s_d^k where k is number of multiplier, d is the first index of first multiplier; serves a potential-growth indicator with the above-stated property.

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References

[1] Svirezhev, Yu.M. & D.O. Logofet , 1978, Stability of biological communities, *Nauka*, Moscow, Russia (in Russian); English translation: *Mir*, Moscow, Russia, 1983.

[2] Logofet, D.O. & I.N. Klochkova, 2002, Mathematics of the Lefkovitch model: the reproductive potential and asymptotic cycles *Matematicheskoe modelirovanie.* (*Mathematical modelling*), 10, 116-126 (in Russian).