On circulant populations: the algebra of semelparity.

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The *semelparous species* are those whose individuals reproduce only once in their life and then die. Among these species there are some plants, fishes (salmon) and many insects. The reproduction happens simultaneously (in the spring) and is concentrated in a very short period. The population of such species can be subdivided into age/year classes. The age classes compete with each other for a shared resource. As a result one or several year classes can go extinct. It can even happen that only one of all year classes survives the competition. If this is the case, we say that the population exhibit Single Year Class (SYC) behaviour [3].

The most striking example of SYC cycles are those of cicada. Magical cicadas live 13 or 17 years under the ground, sucking juice of tree roots. After that they emerge all together, they fly, they sing, they mate, producing a lot of noise and eggs, and fall on the ground to die (if they escaped the fate to be eaten by birds being lucky to participate in a feast of fat cicadas). The cicadas have been given the name "magical" for their noisy and rare (but periodical) appearance. But where do these "magical" large prime numbers: 13 and 17 come from? The question is still open [4, 5].

We consider a matrix model of semelparous species dynamics for a population consisting of k age classes. The model is invariant under a cyclic shift of indices (an index denotes a number of an age class). We call such a population *circulant* [1]. Interactions between individuals are introduced via an "environmental" variable (which is in our case a one-dimensional quantity). The age classes differ by their impact on and sensitivity to the environment.

The system exhibits two basic types of behaviour: coexistence of the year classes or competitive exclusion between them. We try to find conditions to have one of these two types. The problem is rather completely solved for biennials ($k =$

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2) and the main result can be formulated as follows. Competitive exclusion occurs if sensitivity increases with age, while impact decreases with age. It also occurs if sensitivity decreases with age, while impact increases sufficiently strongly with age [2].

The case $k = 3$ allows for another interesting type of behaviour as competitive exclusion with sudden switches between year classes mathematically, it corresponds to heteroclinic 3-cycle). For larger values of k the main question is "is it possible to have more than one year class present than others are missing?".

Mathematically, switches between two main types of behaviour are described as degenerate bifurcations we call them "vertical bifurcations") when an whole hypersurface consisting of k-cycles occurs in the phase space. Corresponding conditions on impacts and sensitivities have a form for corresponding circulant matrices to be singular.

Another approach to the system is to introduce a small parameter (we consider values of the basic reproduction ratio R_0 slightly above 1) and reduce it to a system of ODE's, which is more amenable to an analysis. But then a question arises: which of the conclusions, we have found for the system of ODE's, are still valid for the original matrix model?

References

[1] Davydova, N. V., Diekmann, O., Gils, S. A. van: On circulant populations. I. The algebra of semelparity. Submitted to Lin. Algebra Appl.

[2] Davydova, N. V., Diekmann, O., Gils, S. A. van: Year Class Coexistence or Competitive Exclusion for Strict Biennials? J. Math. Biol., **46**, 95–131 (2003)

[3] Davydova N. V.: Dynamics and bifurcations in families of single year class maps. Submitted to SIAM J.Appl.Dyn.Sys.

[4] Behncke, H.: Periodical cicadas. J. Math. Biol., **40,** 413–431 (2000)

[5] Webb, G. F.: The prime number periodical cicada problem. Discrete Contin. Dyn. Syst. Ser. B., **1**, 3, 387–399 (2001)