Dispersal in Predator-Prey Systems

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Distributions of dispersal times are incorporated into Lotka-Volterra models. These are formulated as integro-differential equations that describe predator-prey dynamics and dispersal between habitat patches. The two-patch Lotka-Volterra system for the prey density $N(T)$ and predator density $P(T)$ at time T, then takes the form:

$$\frac{dN_i(T)}{dT} = (R-AP_i(T))N_i(T)+D_N[\int_0^\infty G_N(S)e^{-M_NS}N_j(T-S)dS-N_i(T)],$$

$$\frac{dP_i(T)}{dT} = (BN_i(T)-M)P_i(T)+D_P[\int_0^\infty G_P(S)e^{-M_PS}P_j(T-S)dS-P_i(T)].$$

for $i,j = 1,2$ and $i \neq j$, with subscripts indicating the patch number. Here positive parameters R, M, A, and B describe the growth of the prey, the decline of the predator, the linear functional and numerical responses, respectively. Probability density functions $G_N(S)$, $G_P(S)$ are defined for the time that it takes a prey, predator to disperse (given that the individual survives the trip), and M_N, M_P are the constant probabilities per unit time for the prey, predator disperser to perish while travelling. The emigration rates for prey and predator are denoted by D_N and D_P, respectively.

If one species disperses between habitat patches (predators are often more mobile than their prey), then linearization in the neighborhood of the spatially homogeneous positive equilibrium and examination of the characteristic equation shows that the dispersal almost always stabilizes this equilibrium. The exception occurs when every trip has exactly the same duration, thus the travel time distribution is a delta function. In this case of discrete delays, there is a set of parameter values with zero measure for which the linearization method used is inconclusive. The same conclusions hold for one species dispersal in configurations of multiple patches.

The stabilizing effect of dispersal has previously been found for patches coupled through a pool of dispersers using ordinary differential equation models by Holt (*Am.Nat.* 124, 377-406, 1984), Weisser & Hassell (*Proc. R. Soc. London B* 263, 749-754, 1996) and Weisser et al. (*J. Theor. Biol.* 189, 413-425, 1997). This effect continues to hold when the dispersal is modelled more realistically using probability density functions as above.

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