

## A mathematical model of blood production involving time-delay

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We study a mathematical model of hematopoiesis, the production of blood cells in the bone marrow. The model takes the form of a system of two nonlinear differential equations involving time delay. In this model, the delay corresponds to the time required for a cell to divide.

This model has been introduced by Mackey in 1978 [2] as a delay differential equation. In 1994, Mackey and Rudnicki [3] considered a more general model by taking into account a cell cycle with two phases, a proliferating and a resting phase. The model of Mackey and Rudnicki [3] has been studied by Dyson *et al* in 1996 [1] and Mackey and Rudnicki in 1999 [4].

We consider a model with two phases, as in [3]. However, we suppose that all cells do not divide, in the proliferating phase, at the same age, but divide according to a density function with a compact support. We study the stability of the system by using Liapunov functionals.

### References

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