Asymptotic behaviour of structured metapopulation models

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The habitat of many populations is fragmented in separate "patches" connected through migration. Some suitable patches may remain empty, either because of local "catastrophes" or because of chance extinctions. This structure is known in the biological literature as a "metapopulation".

Levins [4] presented a very simple and influential model. Assuming that there is an infinite number of patches, each of which is classified simply as empty or occupied, the variable $p(t)$ represents the fraction of occupied patches, and its dynamics is described by the single equation

$$p'(t) = cp(t)(1 - p(t)) - ep(t) \tag{1}$$

where c and e are the (per patch) colonization and extinction rates.

Among the extensions of the model, we consider here the 'structured metapopulation model' [3], in which each patch (instead of being considered either empty or occupied) is classified by its population size: the variables $p_i(t)$ represent the fraction of patches with i individuals, and their dynamics is described by the (infinite) system of differential equations:

$$\begin{cases} p_i'(t) &= -[(b_i + d_i + \gamma_i)i + \rho D(t) + \nu_i]p_i(t) + [b_{i-1}(i-1) \\ & \quad +\rho D(t)]p_{i-1}(t) + [d_{i+1} + \gamma_{i+1})(i+1)]p_{i+1}(t), \quad i \geq 1 \\ p_0'(t) &= -[\rho D(t)]p_0(t) + [d_1 + \gamma_1]p_1(t) + \sum_{i=1}^{\infty} \nu_i p_i(t) \end{cases} \tag{2}$$

where b_i, d_i and γ_i are the (per capita) birth, death and emigration rate, while ν_i is the (per patch) catastrophe rate. $D(t) = \sum_{i=1}^{\infty} \gamma_i i p_i(t)$ is the total dispersal rate, and ρ is the probability of successful immigration of a disperser. The main restrictive assumption in this model is that immigration is the same ($\rho D(t)$) in all patches, independently of spatial structure.

We show [1], under rather general assumptions, that this system of equation always converges to an equilibrium, the extinction equilibrium below a threshold, an endemic equilibrium with a distribution of population sizes among patches, above the threshold. The threshold can be described as a 'reproduction number': the expected number of successful dispersers from a newly colonized patches before its extinction must be larger than 1 (see also [2]).

Furthermore, we discuss the conditions under which the endemic equilibrium distribution can be described through Levins' model. In words, these are that each patch has a similar carrying capacity; second, that the average time between catastrophes should be larger than the time necessary to reach the carrying capacity from an initial state consisting of just one individual, but smaller than the time in which extinction starting from the carrying capacity can occur through 'demographic fluctuations'.

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