Evolutionary games with variable payoffs

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Matrix games are commonly used to model animal populations because they are both simple and generate meaningful results. The game is defined by a set of strategies, and a corresponding matrix of payoffs. Matrix games have been used to model a variety of animal behaviours. They are structurally and relatively straightforward and thus amenable to analysis, but at the same time provide plausible, if simplistic, explanations for certain behaviours. A matrix may possess a unique ESS, no ESSs or many ESSs. See [1],[2] for a discussion of the possible complexity of the ESS structure of a matrix.

It is almost universally assumed that the payoffs in the payoff matrix are independent of time. This corresponds with the payoff matrix of constants, A. In the real world, however, the time at which contests occur can be crucial. As the breeding season develops there is a natural variation in the rewards available for any given contest [6]. Environmental changes may also cause variations from year to year, and even day to day due to unpredictable effects such as the weather [5].

In this presentation we introduce the variable payoff matrix $A(t)$ to consider matrix (and bimatrix) games where the payoffs are functions of time and investigate how different time-dependent payoffs may affect strategies. In particular we consider the relationship between the payoff matrix and the mean population state as each vary in time, the population state at any time being found using the classical replicator dynamic [3], [4].

Rules are found which hold in this more general situation, and the complexity of possible behaviour is underlined by demonstrating other plausible conditions which do not hold. An application of the ideas introduced in the presentation is illustrated using a game where individuals have two successive decisions to make.

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