Numerical study of a structured population model in an environment with a dynamical food-source

Oscar Angulo[1] and J.C. López-Marcos[2] .

We will study the numerical integration of a nonlinear model that describes the dynamics of a size-structured population feeding on a dynamical food-source. Such model is determined by the next equations:

$$u_t + (g(x, S(t))\, u)_x = -\mu(x, S(t))\, u, \quad 0 < x < 1,\ t > 0, \tag{1}$$

$$g(0, S(t))\, u(0, t) = \int_0^1 \alpha(x, S(t))\, u(x, t)\, dx\ , \quad t > 0, \tag{2}$$

$$u(x, 0) = \phi(x), \quad 0 \le x \le 1, \tag{3}$$

$$S'(t) = f(t, S(t), I(t)), \quad t > 0, \qquad S(0) = s_0, \tag{4}$$

$$I(t) = \int_0^1 \gamma(x, S(t))\, u(x, t)\, dx, \quad t > 0. \tag{5}$$

The independent variables x and t represent, respectively, size and time, and the function $u(x, t)$ is the population density of individuals with size x at time t. The population dynamics is determined by the growth rate g, the mortality rate μ and the fertility rate α. These vital functions depend on the structuring variable and on the available food resources that are given by the function $S(t)$. Also, the dynamics of such resources is bound to the distribution of the population as equations (4) and (5) reflect.

The present work approaches the numerical integration of equations (1)-(5) using a generalization of the scheme presented in [1]. We will analyze

[1]Departamento de Matemática Aplicada a la Técnica, Escuela Universitaria Politécnica, Universidad de Valladolid, C/ Francisco Mendizábal 1, 47014 Valladolid, Spain (e-mail: oscar@mat.uva.es).

[2]Departamento de Matemática Aplicada y Computación, Facultad de Ciencias, Universidad de Valladolid, Valladolid, Spain (e-mail: lopezmar@mac.uva.es).

the behaviour of such numerical scheme by means of the study of the results obtained in the integration of a problem that describes the dynamics of ectothermic invertebrates, e.g., the water flea *Daphnia magna*, feeding on a dynamical algal population [2, 3].

References

[1] O. Angulo. *Estudio Numérico de Modelos de Poblaciones Estructuradas Por el Tamaño*. PhD thesis, Universidad de Valladolid, 2002.

[2] S.A.L.M. Kooijman & J.A.J. Metz, On the dynamics of chemically stressed populations: the deduction of population consequences from effects on individuals, *Ecotox. Env. Safety*, 8: 254–274, 1984.

[3] A.M. de Roos. Numerical methods for structured population models: The escalator boxcar train. *Numer. Methods Partial Differential Equations*, 4:173–195, 1988.