

Multiple deterministic skeletons and lattice effects in stochastic population models

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The dynamics of stochastic models, though often strikingly different from their deterministic counterparts, can nevertheless frequently be understood with reference to transients or unstable invariant sets in deterministic models. I will illustrate this point with a few examples. In general, the process of adding noise to a deterministic model is a well-defined recipe for creating a stochastic model, but the inverse procedure, of “subtracting” the noise from a stochastic model, is not well-defined. The upshot is that there can be multiple “generalized deterministic skeletons” corresponding to a given stochastic model. Although in cases of interest the differences among these may be small, but the small differences can have surprising consequences.

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