Choosing a noise term in single-species population models

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Discrete, single-species population models with first order feedback have, despite their simplicity, successfully been used to model the dynamics of such diverse populations as aphids, sandhill-cranes and human beings [1]. The models are often deterministic and fitted to data using the method of least squares. While deterministic population models are often sufficient, there are several reasons for considering a stochastic population model. Stochastic forces may have a profound impact on the population dynamics, time to extinction can be estimated and a stochastic population model can be fitted using the method of maximum likelihood.

When stochastic population models have been fitted to data, these have traditionally been constructed [2] by either assuming the noise to be normally distributed with constant variance around the expectation

$$a_{t+1} = f(a_t) + \sigma\epsilon_t$$

where $\epsilon_t \sim \mathrm{N}(0,1)$, a_{t+1} is the population in generation $t+1$ and $f(a_t)$ the expectation, or around the logarithm of the expectation

$$\log a_{t+1} = \log f(a_t) + \sigma\epsilon_t - \frac{\sigma^2}{2}$$

Both models are primarely chosen for mathematical convenience rather than biological reasons. Arguing theoretically, based on reasoning on how local interactions produce population dynamics, we propose two new stochastic models, where the variance around the expectation $f(a_t)$ is proportional to $\sqrt{f(a_t)}$ when most of the noise is demographic [3], and proportional to $f(a_t)$ otherwise. These are then compared with other models of stochasticity previously used in the literature.

Using the method of maximum likelihood we fit several stochastic models to short time-series from an individual based model [4] with overcompensatory dynamics. The results show that the assumptions made about randomness have a profound impact on parameter estimation and likelihood when the dynamics are oscillatory or chaotic.

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References

[1] A. A. Berryman. *Principles of Population Dynamics and Their Applications.* Stanley Thornes Ltd, 1999.

[2] R. Hilborn and M. Mangel. *The Ecological Detective.* Princeton University Press, 1997.

[3] S. Engen, Ø. Bakke, and A. Islam. Demographic and environmental stochasticity–concepts and definitions. *Biometrics*, 54:840–846, 1998.

[4] A. Johansson and D. J. T. Sumpter. From local interactions to population dynamics in site-based models of ecology. *Theoretical Population Biology*, 2003 (in press).