Synchronized fade-outs of epidemic outbreaks in networks of cities

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Recurrent epidemics of infectious childhood diseases such as measles are a major health problem and have been subject to extensive theoretical research. Here we address the problem of epidemic 'fade-outs' when in some years major outbreaks are unexpectedly suppressed, and the disease fails to spread on a large scale. We develop a theory for the dynamics of fade-outs and their synchronization to the seasonal cycle in a network of coupled cities.

A system of coupled seasonally forced SEIR models is used to explore the epidemic dynamics in a network of interacting cities. In the parameter range typical for measles, the models generate chaotic dynamics and for each city the strange attractor in phase space has a characteristic "toothlike" structure with annual and biennial dynamics in excellent agreement with established long-term data sets. A new qualitative criterion based on the attractor topology is developed to distinguish between major outbreaks and epidemic fade-outs. This information is coded in a symbolic dynamics scheme which comprises all information about the timing of epidemic fade-outs.

We are able to deduce a one dimensional first return map of the chaotic SEIR equations which describes successive minima of susceptibles. By

iteration of the return map the symbolic sequence of fade-outs can be retrieved. This allows us to understand and predict fade-outs in terms of explicit epidemiological parameters.

The results are compared to real data by first reconstructing the number of susceptibles from data records of infected incidences. When applied to classical measles data sets the reconstructed attractor strongly resembles the "tooth attractor" of the SEIR model and confirms the existence of a first return map.

Finally we address the synchronization of epidemic fade-outs in a network of interacting cities. Synchronization is defined as measure based on the symbolic dynamic sequence of a group of cities. This is applied to real data sets and numerical simulation results for different network topologies. The relevance of synchronized fade-outs is discussed in the context of the global persistence of epidemics and in light of the so-called 'rescue effect.'

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