Oscillations in Differential Equations With State-Dependent Delays

K. NIRI¹ .

Recently, O.Arino, M.L.Hbid and R.Bravo de la Parra proposed a model in [1] describing the evolution of a population of fish whose larvae share a limited resource. The model is made of two equations: a state equation governing the evolution of the total number, which is a delay differential equation with variable delay, and a ordinary differential equation satisfied by the delay with coefficients expressed in terms of the state variable. A simplified version of the model which was presented is the following delay differential equations with delay depending directly on the state.

$$\begin{cases} \frac{dx}{dt} = -f(x(t - \tau(t))), \\ \frac{d\tau}{dt} = h(x(t), \tau(t)). \end{cases}$$

In [2], the authors studied the existence of slow oscillations and periodic slow oscillating solutions.

Here, we consider a general abstract formulation of this system :

$$\begin{cases} \frac{dx}{dt} = -f(x(t - \tau(x_t))), \\\\ \int_{t - \tau(x_t)}^t k(x(s))ds = k_o \end{cases}$$

and we extend, for this class of differential equations, results about necessary and sufficient conditions of oscillations which are well known in

the case when the delay is constant, and in some classes of time-depending delay differential equations

References

- O.Arino,M.L.Hbid and R.Bravo de la Parra, A Mathematical model of population of fish in the larval stage: density-dependence effects.Math.Biosci.150,1998,1-20
- [2] O.Arino,K.P.Hadeler and M.L.Hbid, Existence of periodic solutions for delay differential equations with state dependent delay.J.Diff.Equat.144,N2,1998,263-301

¹Département de Mathématiques et Informatique, Faculté des Sciences. Université Hassan II-Aïn chock, Casablanca, Maroc (e-mail:).