# Partial Functional Mathematical Model of Nitrogen Transformation Cycle

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We consider a system of parabolic equations with discret time delays describing nitrogen transformation cycle in water environment , which takes the form:

$$\frac{\partial x_i(p,t)}{\partial t} = d_{xi}\frac{\partial^2 x_i(p,t)}{\partial p^2} + x_i(p,t)U_i(x(p,t)) -$$

$$x_i(p,t-r_{ei})E_i(x(p,t-r_{ei})) - x_i(p,t-r_{mi})M_i(x(p,t-r_{mi})))$$

$$\frac{\partial x_5}{\partial t} = d_{x5}\frac{\partial^2 x_5}{\partial p^2} + \sum_{i=1}^{4} x_i(p,t-r_{mi})M_i(x(p,t-r_{mi}) - K_5 x_5(p,t-r_5)$$

$$\frac{\partial x_6}{\partial t} = d_{x6}\frac{\partial^2 x_6}{\partial p^2} + K_5 x_5(p,t-r_5) - U_1(x)x_1 - P_6(x)x_4 + E_4(x(p,t-r_{e4}))x_4(p,t-r_{e4})$$

$$\frac{\partial x_7}{\partial t} = d_{x7}\frac{\partial^2 y_1}{\partial p^2} + E_1(x(p,t-r_{e1}))x_1(p,t-r_{e1}) + U_2(x)x_2 - P_7(x)x_4$$

$$\frac{\partial x_8}{\partial t} = d_{x8}\frac{\partial^2 y_i}{\partial p^2} + x_2(p,t-r_{e2})E_2(x(p,t-r_{e2})) - x_3 U_3(x)$$

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$$\frac{\partial x_9}{\partial t} = d_{x9}\frac{\partial^2 y_i}{\partial p^2} + x_3(p,t-r_{e3})E_3(x(p,t-r_{e3})) - x_4 P_9(x)$$

where $x_i, i = 1,...,9$, are the concentration of the recycling nitrogen in microorganisms,phytoplankton, detritus, DON, ammonium, nitrites, nitrate. From the biological viewpoint the functions $F_i$, $U_i$, $E_i$ and $M_i$ describe the growth, uptake, excretion and mortality rate of the living organisms, respectively.

The purpose of the paper is to study the dynamic property of the parabolic system in relation to its corresponding elliptic system. We also investigate the dynamics of the coupled parabolic-ordinary system. Stability analysis of equilibria is given.

## References

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