Attenuant cycles of concave population models with periodically fluctuating environments

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It is known that a positive rest point $x = K$ of the following Beverton-Holt equation is globally attractive:

$$x_{t+1} = \frac{\lambda}{1 + (\lambda - 1)(x_t/K)} x_t, \quad x_0 \in [0, +\infty), \ \lambda > 1, \ K > 0.$$

The equation can have a periodic solution if the constant parameter K is replaced by the periodic sequence K_t. That is, the following non-autonomous version of the Beverton-Holt equation can have a periodic solution:

$$x_{t+1} = \frac{\lambda}{1 + (\lambda - 1)(x_t/K_t)} x_t, \quad x_0 \in [0, +\infty), \ \lambda > 1, \ K_t > 0, \qquad (1)$$

where $\{K_t\}_{t \geq 0}$ is a periodic sequence with positive elements. Cushing and Henson [2] showed that if $\{K_t\}_{t \geq 0}$ is periodic with a base period 2, then Eq.(1) has a globally attractive periodic solution $\{p_1, p_2\}$ with a base period 2. Furthermore, it was shown that the periodic solution $\{p_1, p_2\}$ is *attenuant*, i.e the following inequality holds:

$$\frac{p_1 + p_2}{2} < \frac{K_1 + K_2}{2}.$$

This implies that the environmental fluctuation is deleterious to a population in the sense that its time average of the population density in fluctuating environment is less than that in a constant environment with the same average.

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Whether the Beverton-Holt equation with $\{K_t\}_{t \geq 0}$ whose base period is greater than 2 has a globally attractive periodic solution is an open problem. Furthermore, attenuation of the periodic orbit is not investigated (see Cushing and Henson [3]). In this presentation, I would like to consider these problems. I would also like to show what kind of property of population models is sufficient for attenuation of a periodic solution since the attenuation depends on models (Cushing [1] showed that other population models can have the opposite property). Moreover, I would also like to discuss the relationship between attenuation observed in the population models with a periodically fluctuating environment caused by the external force and by the effect of density dependence (see Kon [4]).

References

[1] Cushing, J. M.: Oscillatory population growth in periodic environments, *Theoretical Population Biology* **30**, 289-308 (1986)

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[4] Kon, R.: Permanence of discrete-time Kolmogorov systems and saturated rest points, (preprint)