Evolution of sequential hermaphroditism in a structured population model

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The classical models of Sharpe-Lotka-McKendrick (linear) and Gurtin-MacCamy (non-linear) for the age-structured population dynamics do not explicitly take sexual reproduction into account, e.g. males are always abundant enough to fertilize all the females. Following this work we have derived some models of sexually reproducing populations and, in addition, we have analyzed the *sex-ratio* (proportion between females and males) from an evolutionary point of view. One of them is a model of sequential hermaphroditism where the individuals are born females but become males when they reach a critical size (say, a critical age). The system takes the form of a partial differential equation with a non-linear boundary condition:

$$u_t + u_a + \mu(a,P)\,u = 0\,,$$
$$u(0,t) = \int_0^\infty \beta(x,P)\,(1-s(x))\,u(x,t)\,dx\;\frac{\int_0^\infty \gamma(x,P)\,s(x)\,u(x,t)\,dx}{1+h\int_0^\infty s(x)\,u(x,t)\,dx}\quad,$$

where $P=\int_0^\infty u(x,t)\,dx$ is the total population and we assume that they change from female to male at a random age with a fixed probability distribution function $s(x)$. Conditions for the existence and stability of non-trivial steady states are given.

Assuming that a stable equilibrium is reached $u^*(a)$, we carried on the evolutionary dynamics study for the critical age. So, we have written the linear system for an invader $u^{i)}$ with a different distribution function $s_i(x)$ (function-valued trait):

$$u_t^{i)} + u_a^{i)} + \mu_*(a)\,u^{i)} = 0\,,$$
$$u^{i)}(0,t) = \frac{\int_0^\infty \beta_*(x)\,(1-s_i(x))\,u^{i)}(x,t)\,dx}{2\int_0^\infty \beta_*(x)\,(1-s(x))\,\Pi_*(x)\,dx} + \frac{\int_0^\infty \gamma_*(x)\,s_i(x)\,u^{i)}(x,t)\,dx}{2\int_0^\infty \gamma_*(x)\,s(x)\,\Pi_*(x)\,dx}\,,$$

and we have found a critical age $\hat{l}$ such that the evolutionarily stable strategy (ESS) consists in changing sex at age $\hat{l}$ with probability one. This computation is based on the maximization of a linear functional in a compact convex subset of L^1 whose extreme points are the characteristic functions $\mathcal{X}_{[l,\infty)}(x)$, $l \geq 0$.

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References

[1] Calsina, A., Ripoll, J.: Hopf bifurcation in a structured population model for the sexual phase of monogonont rotifers, J.Math.Biol., **45**, 22-36 (2002)

[2] Charnov, E.L.: *The Theory of Sex Allocation*, Princeton University Press, Princeton 1982

[3] Geritz, S.A.H., Kisdi, É., Meszéna, G., Metz, J.A.J.: Evolutionarily singular strategies and the adaptive growth and branching of the evolutionary tree, Evol. Ecol., **12**, 35-57 (1998)

[4] Webb, G.F.: *Theory of nonlinear age-dependent population dynamics*, Pure and Applied Mathematics, Marcel Dekker Inc., New York and Basel 1985