Mathematical modelling in population dynamics through the class of controlled branching processes

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Branching processes theory provides appropriate mathematical models for description of the probabilistic evolution of systems whose components (cells, particles, individuals in general), after certain life period, reproduce and die. Therefore, it can be applied in several fields (biology, demography, ecology, epidemiology, genetics, medicine,...). In particular, if in population dynamics some control on their size is required, then the controlled branching processes introduced in Sevast'yanov and Zubkov (1974) constitute an interesting class of models to be considered. For example, when it is necessary to control the size in animal populations, a reasonable methodology could be to achieve the corresponding control on the number of female individuals (see del Puerto et al. (2001)). Let us denote by f_{ni} the number of female descendants originated by the i-th female in the generation n, we assume that f_{ni}, for $n = 0, 1, \dots;\ i = 1, 2, \dots$ are i.i.d. random variables. Then, a logical expression for the total number of females in the $(n+1)$-th generation could be:

$$F_{n+1} = \sum_{i=1}^{\phi(F_n)} f_{ni}, \quad n = 0, 1, \dots \tag{1}$$

where the empty sum is considered to be 0, initially $F_0 = f_0 \in Z^+$ and $\phi : R^+ \to R^+$ is a deterministic function assumed to be integer-valued on

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integer arguments. This function will determinate the control of the size in the female population. In fact, when $\phi(F_n) < F_n$ then $F_n - \phi(F_n)$ females are removed, if $\phi(F_n) > F_n$ then $\phi(F_n) - F_n$ females are introduced in the population and when $\phi(F_n) = F_n$, no control is applied on the female population. The stochastic model (1) is precisely the discrete time controlled branching process with deterministic control function. From a practical point of view, an important problem to study is the determination of estimators for the main parameters of the probability distribution associated to f_{ni}, called offspring probability distribution. Recently, this question has been considered in González et al. (2003) and nonparametric estimators have been derived. In this paper, we continue this research and, from a bayesian outlook, parametric estimators for the mean and variance of the offspring probability distribution are determined. In particular, for some classical offspring probability distributions (Poisson, geometric, binomial and negative binomial distribution) the explicit expressions for the proposed estimators are obtained and, through simulated examples, the limit behaviour of their estimates is showed.

References

[1] M. González, R. Martínez and I. del Puerto. Nonparametric estimation of the offspring distribution and the mean for a controlled branching process. *Preprint n: 74. Department of Mathematics. University of Extremadura*, 2003.

[2] I. del Puerto, R. Martínez and M. Molina. Procesos de ramificación de Galton-Watson controlados: Aplicación a poblaciones animales. *Proceeding of the 8th Spanish Conference of Biometry*, 2001, 1, 285-286.

[3] B. A. Sevast'yanov and A. Zubkov. Controlled branching processes. *Theory Probability and their Applications*, 1974, 19, 14-24.