The bisexual branching process with population-size dependent mating as a mathematical model to describe phenomena concerning to inhabit or re-inhabit environments with animal species

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Initially branching processes theory was motivated to explain the extinction phenomenon of aristocracy family lines in the European society. After 1940, the interest in this models increased due to its applications in the physical and biological sciences, mainly to nuclear chain reactions and particles cascades. Recently, ecological applications have been considered by Bruss and Slavchova-Bojkova (1999) and Slavchova-Bojkova (2000), where some modified asexual branching models, the Bienaymé-Galton-Watson process and the age-dependent branching process, respectively, have been used to investigate the probabilistic behaviour of some interesting ecological random variables. In this paper, under a sexual reproduction context, similar problems are studied. We consider phenomena concerning to inhabit or re-inhabit environments with animal species. As mathematical model we consider the bisexual branching process with population-size dependent mating introduced in Molina et al. (2002). This branching model is a two-type discrete time stochastic process defined in the form:

$$(F_{n+1}, M_{n+1}) = \sum_{i=1}^{Z_n} (f_{ni}, m_{ni}),\ Z_{n+1} = L_{Z_n}(F_{n+1}, M_{n+1}),\ n = 0, 1, \ldots \tag{1}$$

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where the empty sum is considered to be $(0, 0)$, $Z_0 = N \geq 1$, (f_{ni}, m_{ni}), $n = 0, 1, \ldots; i = 1, 2, \ldots$, are i.i.d. non negative, integer valued random variables, and $\{L_k\}_{k\geq 0}$ is a sequence of mating functions, i.e. for each $k \geq 1$, $L_k : R^+ \times R^+ \longrightarrow R^+$ is assumed to be monotonic nondecreasing in each argument, integer-valued on integer arguments and such that $L_k(x, y) \leq xy$. Intuitively, (f_{ni}, m_{ni}), represents the number of females and males produced by the i-th mating unit in the n-th generation and, from (1), (F_{n+1}, M_{n+1}) will be the total number of females and males in the $(n + 1)$-th generation. These females and males form $Z_{n+1} = L_{Z_n}(F_{n+1}, M_{n+1})$ mating units, which reproduce independently from each other. This mathematical model can describe the probabilistic evolution of two-sex animal populations with sexual reproduction in which, it is reasonable to allow an individual's mating behaviour to depend on the population-size, e.g. by environments changes or another factors it is possible that the same number of females and males gives rise to different numbers of mating units in different generations.

References

[1] F. Bruss and M. Slavtchova-Bojkova. On waiting times to populate an environment and a question of statistical inference. *Journal of Applied Probability*, 1999, 36, 261-267.

[2] M. Molina, M. Mota and A. Ramos. Bisexual Galton-Watson branching process with population-size dependent mating. *Journal of Applied Probability*, 2002, 39, 479-490.

[3] M. Slavtchova-Bojkova. Computation of waiting time to successful experiment using age-dependent branching model. *Ecological Modelling*, 2000, 133, 125-130.