Demography and dispersal: invasion wave speed and its perturbation analysis

Hal Caswell[1] and Michael G. Neubert[2] .

When a population successfully invades a new region, it increases in abundance and spreads in space. Under idealized conditions, the asymptotic rate of spread is a constant c^*, known as the invasion wave speed. The wave speed depends on both demography (the stage-specific rates of survival, growth, maturation, reproduction, etc.) and dispersal (the stage-specific probability distribution of movement distances). The wave speed can be calculated from a model that combines a matrix population model for demography with an integrodifference equation for dispersal.

Let $\mathbf{n}(x,t)$ be the population vector at location x and time t, and $\mathbf{A}$ the population projection matrix evaluated at low density. Movement is described by a dispersal kernel matrix $\mathbf{K}(x,y)$ whose (i,j) entry, $k_{ij}(x,y)$, is the probability density of the location x at time $t+1$ of an individual that started at location y at time t, and that made the transition from from stage j to stage i during the interval $(t,t+1]$. We will assume that the probability of dispersing from y to x depends only on the relative locations of y and x; thus $\mathbf{K}(x,y) = \mathbf{K}(x-y)$. The resulting model is

$$\mathbf{n}(x,t+1) = \int_{-\infty}^{\infty} [\mathbf{A} \circ \mathbf{K}(x-y)]\, \mathbf{n}(y,t)\, dy \qquad (1)$$

where $\circ$ denotes the Hadamard product.

These models lend themselves to construction from real demographic and dispersal data, and the resulting estimates of c^* play a role analogous to the population growth rate λ in demography.

[1]Biology Department MS-34, Woods Hole Oceanographic Institution, Woods Hole MA 02543, USA (e-mail: hcaswell@whoi.edu).

[2]Biology Department MS-34, Woods Hole Oceanographic Institution, Woods Hole MA 02543, USA (e-mail: mneubert@whoi.edu).

Of particular importance is the perturbation analysis of c^*. We will show how to calculate the sensitivity and elasticity of c^* to changes in (1) any of the demographic parameters and (2) any parameter characterizing the distribution of dispersal distances. Animportant special case occurs when the dispersal data consist of measurements of the dispersal distances of a sample of individuals. The distribution can be characterized by its order statistics and the sensitivity and elasticity of c^* to each order statistic calculated. These results document the importance of long-distance dispersal, even when it is rare, to invasion wave speed.