Two Optimal Control Problems Related to the Wastewater Disposal Problem

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Pollution of marine environment is caused by the adverse impact of domestic, industrial or agricultural contaminants released into the sea by means of wastewater discharges from sewage farms, where biological and/or chemical processes are previously used to reduce contaminant concentrations. The usual sources occur near urban settlements and industrial developments. For urban and industrial sewage the contaminant source is generally through a direct discharge from a long sea outfall.

We consider a domain $\Omega \subset R^2$ occupied by shallow waters (for instance a lake, an estuary or a *ría*), and we assume that N submarine outfalls dump an amount $m_j(t)$ of sewage into Ω. Each of the outfalls will be located at a point b_j in Ω, and it is connected to a corresponding sewage farm (located at a fixed point a_j) by a pipeline. We also suppose that there exist M areas $A_i \subset \Omega$ (for instance fisheries, beaches or recreation zones), where it is necessary to guarantee the water quality with pollution levels lower than maximum thresholds. With this aim in view, several parameters can be

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considered (dissolved oxygen, heavy metals concentration, temperature, pH, radioactivity...), all of them indicating water quality. In the case of domestic discharges, the most important are the *Dissolved Oxygen* (DO) and the *Biochemical Oxygen Demand* (BOD). In order to avoid sanitary problems, we have to guarantee a minimum level ζ_i of DO and a maximum level σ_i of BOD in each region A_i to be protected.

We are interested in optimizing the wastewater treatment system in two ways: obtaining the optimal depuration levels in the discharges [3,4], and finding the optimal location of the wastewater outfalls [1,2], which yield the lowest economical cost of the system, also guaranteeing the above mentioned constraints on the water quality. Both problems can be formulated as pointwise optimal control problems with state and control constraints.

In order to obtain the numerical solution of the problems we discretize them by using a characteristics-Galerkin method. Finally, the discrete optimization problems are solved by different methods: an interior point algorithm, a duality algorithm, and the Nelder-Mead simplex method.

References

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