Dynamical study of an activated sludge processes model

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A model of wastewater treatment is presented as a nonlinear ordinary differential system. The model describes an activated sludge process in which the substrate is removed by one suspended bacterial population. We assume that the aerator is sufficiently aerated to provide the dissolved oxygen necessary for the growth of bacterial population. Three phenomena are considered. First the reaction kinetics in the aerator linked to microbial growth, substrate degradation and recycled biomass from the settler. The oxygen reaction is neglected. The mass balance of the various constituents gives the following set of equations;

$$(S1) \quad \begin{cases} \dot{s} = -\dfrac{\mu(s)x}{Y} - (1+r)Ds + Ds_{in} & ; \quad s(0) = s_0 \\ \dot{x} = \mu(s)x - (1+r)Dx + rDx_r & ; \quad x(0) = x_0 \\ \dot{x}_r = \nu(1+r)Dx - \nu(w+r)Dx_r & ; \quad x_r(0) = x_{r0} \end{cases}$$

where s, x and x_r are the states variables representing the substrate, the biomass and recycled biomass concentrations, respectively.
For this model, we study the basic properties of invariance, dissipation and persistence and we prove that under a condition on parameters, there exists an interior equilibrium point which is globally asymptotically stable. Furthermore, by using properties of cooperative dynamical systems, we prove that the model is robust, that is, when the growth rate function is not well known, but only its upper and lower bounds, we prove that there is an invariant interior cube (in the three dimensional space (s, x, x_r)). Finally, we show that with feedback control of the recycled rate r, we can reduce the invariant cube to a small volume around a set point.

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