Feedback Control for Competition Models with Inhibition in the Chemostat

Gonzalo Robledo[1] and Jean Luc Gouzé[2] .

We consider the feedback control problem for a model of competition among two microorganisms with one limiting resource in the chemostat with nonmonotone uptake functions described by the system of differential equations:

$$\begin{cases} s' = D(s_{in} - s) - \frac{x_1}{y_1} f_1(s) - \frac{x_2}{y_2} f_2(s) \\ x_1' = x_1(f_1(s) - D) \\ x_2' = x_2(f_2(s) - D) \end{cases} \quad (1)$$

In model (1), $s(t)$ denotes the concentration of limiting resource and x_i denotes the density of the ith population of microorganisms at time t, $f_i(s)$ is a nonmonotone and unimodal function (*e.g.* a Haldane type function) and represents the case when the limiting resource is essential at low concentrations but may be inhibiting or toxic at higher concentrations for the growth of the ith population, y_i is a growth yield constant; D and s_{in} denote, respectively, the dilution rate of the chemostat and the concentration of the growth limiting resource.

Competition theory for the chemostat models (see [1] and [4]) predicts that coexistence of species for model (1) is not possible; Control theory allows to obtain coexistence considering limiting resource and density of microorganisms as state variables and dilution rate D or (and) input nutrient s_{in} as control variable(s).

In [2], De Leenheer and Smith study the feedback control of model (1) with monotone functions f_i. In the present work we consider D as the feedback control variable and we suppose that the only output available is the total biomass $x_1 + x_2$, *i.e.* we suppose $D = \alpha g(x_1 + x_2)$ with $\alpha > 0$ and g a continuous and positive real function. We obtain sufficient conditions for the coexistence of microorganisms (obtained as a globally asymptotically stable critical point); Asymptotically autonomous dynamical systems theory and Monotone dynamical systems theory are the main tools employed. Remarks about the robustness of model are shown, if the functions f_i are only known by the lower and upper envelope.

[1] INRIA Sophia Antipolis, Projet COMORE, BP 93 – 06902 Sophia Antipolis Cedex France (e-mail: grobledo@sophia.inria.fr).

[2] INRIA Sophia Antipolis, Projet COMORE, BP 93 – 06902 Sophia Antipolis Cedex France (e-mail: gouze@sophia.inria.fr).

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