Mummy evergreen, why do you keep on growing? Stochastic dynamic programming in theoretical ecology

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Mammals and other organisms present *determinate growth*: they stop growth when they become mature and start to reproduce. But many animals and plants, such as fishes, snakes, clams, etc, experience *indeterminate growth*: their life-history shows trade-offs between growth and reproduction. This latter resource allocation pattern is a theoretical challenging evolutionary problem.

We present a general mathematical model of resource allocation for which we can prove by stochastic dynamic programming that determinate growth is the optimal strategy in a deterministic environment, while a stochastic one leads to optimal strategies with ranges of indeterminate growth.

An individual organism is described by a single quantity, its size x, and depends on a single resource r (food) which is supplied in stochastic quantity at regular time steps, following an independent Markov chain. At each time step, the organism may die but, if it survives, it can mobilize this stochastic resource (the bigger the organism, the more its availability) and the assimilated resource $f(x, r)$ can be allocated either to growth or to reproduction. The lifetime energy allocated to reproduction is individual fitness (contribution to future generations). The trade-off is between full allocation to growth, thus ensuring a better mobilization of resources and hence more offspring in the long run, and full allocation to reproduction, thus ensuring immediate offspring. In a Darwinian perspective the organism with the maximum expected fitness has the "optimal strategy".

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Mathematically, various dynamic programming equations are derived, according to the criterion choice, the state of the environment (deterministic, stochastic and unknown to the organism, or stochastic and observable by the organism), etc. Under assumptions of concavity and/or of growth for the dynamics and for instantaneous and final costs, we prove that optimal strategies are sort of "stochastic control limit" policies, or (σ, Σ) policies found in inventory control. General patterns for the optimal strategies and size trajectories are then interpreted in an ecological perspective and compared with the deterministic case.

References

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