Basic epidemic model: demographic and disease thresholds

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A simple epidemic model incorporating both demographic and epidemiological processes and accounting the possibility of mortality/immigration of susceptible individuals is explored. The model has a ratio-dependent form

$$\frac{d\,x}{d\tau} = \nu\,R_d\,(x+y)\,(1-(x+y)) - \nu\,x - R_0\,\frac{x\,y}{x+y} \equiv X(x,y),$$

$$\frac{d\,y}{d\tau} = -y + R_0\,\frac{x\,y}{x+y} \equiv Y(x,y),$$

where x, y are, correspondingly, normalized numbers of susceptibles and infectives in the population. The following quantities, basic demographic reproductive number (R_d), basic epidemiological reproductive number (R_0), and ratio between average life spans of susceptible and infective classes (ν), are utilized in qualitative analysis.

A global and complete bifurcation analysis is carried out and resulted in the phase-parameter portraits of the model. The non-analytic vector field is handled by a blow-up transformation and a family of homoclinics is found to describe the possible outbreak of disease ignited by a tiny amount of initial infected individuals.

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